USN | | | | | | | | | | |

17CS36

Third Semester B.E. Degree Examination, Jan./Feb. 2023
Discrete Mathematical Structures

Time: 3 hrs. Max. Marks: 100

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Module-1

1 a. Construct the truth tables for the following compound statement :
i) $p \vee (q \wedge r)$ ii) $q \wedge (\sim r \rightarrow p)$. **(06 Marks)**

b. Simplify the following logical statement using laws of logic :
$\sim [\sim [(p \vee q) \wedge r] \vee \sim q]$. **(07 Marks)**

c. Establish the validity of the following argument.

$$\begin{array}{l} p \rightarrow (q \rightarrow r) \\ p \vee \sim s \\ q \\ \hline \therefore s \rightarrow r \end{array}$$
(07 Marks)

OR

2 a. Define Tautology. Prove that, for any propositions p, q, r, the compound proposition $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$ is tautology. **(06 Marks)**

b. Determine the truth value of each of the following quantified statements, the universe being the set of all non – zero integers.
i) $\exists x, \exists y, [xy = 1]$ ii) $\exists x, \forall y, [xy = 1]$ iii) $\forall x, \exists y [xy = 1]$
iv) $\exists x, \exists y, [(2x + y = 5) \wedge (x - 3y = -8)]$
v) $\exists x, \exists y, [(3x - y = 17) \wedge (2x + 4y = 3)]$. **(07 Marks)**

c. If m is an even integers then prove that m + 7 is odd. **(07 Marks)**

Module-2

3 a. Prove by mathematical induction, that
$$1^2 + 3^2 + 5^2 + \ldots\ldots. + (2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}$$ for all integers $n \geq 1$. **(06 Marks)**

b. Define Sum rule and Product rule. Find the number of permutations of the letters of the word M A S S A S A U G A. In how many of these, all four A's are together? How many of them begin with s? **(07 Marks)**

c. A certain question paper contains three parts A, B, C with <u>four</u> questions in <u>part A</u>, <u>five</u> questions in <u>part B</u> and <u>six</u> in <u>part C</u>. It is required to answer seven questions selecting at least two questions from each part. In how many different ways can a student select his seven questions for answering? **(07 Marks)**

OR

4 a. Show that $2^n > n^2$ for all positive integers n greater than 4. **(06 Marks)**

b. How many positive integers n can we form using the digits 3, 4, 4, 5, 5, 6, 7, if we want n to exceed 5, 000000? **(07 Marks)**

c. If F_0, F_1, F_2….. are Fibonacci numbers, prove that $\sum_{i=0}^{n} Fi^2 = F_n \times F_{n+1}$. For all positive integers n. **(07 Marks)**

Module-3

5 a. For any sets A, B, C $\subseteq$ U, show that
i) $A \times (B \cap C) = (A \times B) \cap (A \times C)$ ii) $(A \cup B) \times C = (A \times C) \cup (B \times C)$. **(06 Marks)**

b. Let A = {1, 2, 3, 4, 6} and R be a relation on A defined by a R b iff "a is multiple of b".
i) Write down the relation R and matrix m(R).
ii) Write Digraph and list indegree and outdegree of every vertices. **(07 Marks)**

c. Let A = {1, 2, 3, 4, 5} and define R on A × A by $(x_1\ y_1)\ R\ (x_2\ y_2)$ if and only if $x_1 + y_1 = x_2 + y_2$.
i) Verify that R is an equivalence relation.
ii) Determine the partition of A induced by R. **(07 Marks)**

OR

6 a. Prove that a function $f : A \rightarrow B$ is invertible if and only if it is bijective. **(06 Marks)**

b. Draw the Hasse diagram representing the positive divisors of 36. **(07 Marks)**

c. Let f, g : R → R with $f(x) = ax + b$ and $g(x) = 1 - x + x^2$.
If $(g\ .\ f)(x) = 9x^2 - 9x + 3$, determine a and b. **(07 Marks)**

Module-4

7 a. In how many ways can the 26 letters of the English alphabet be permuted so that none of the patterns CAR, DOG, PUN or BYTE occurs? **(06 Marks)**

b. Determine the number of positive integers n such that $1 \leq n \leq 100$ and n is not divisible by 2, 3 or 5. **(07 Marks)**

c. Solve the recurrence relation $a_n + a_{n-1} - 6a_{n-2} = 0$ for $n \geq 2$ given that $a_0 = -1$ and $a_1 = 8$. **(07 Marks)**

OR

8 a. Solve that recurrence relation $F_{n+2} = F_{n+1} + F_n$ for $n \geq 0$, given $F_0 = 0$, $F_1 = 1$. **(10 Marks)**

b. Determine in how many ways can the letters in the word A R R A N G E M E N T be arranged so that three are exactly two pairs of consecutive identical letters. **(10 Marks)**

Module-5

9 a. Define Konigsberg Bridge Problem with neat sketch. **(06 Marks)**

b. Define Complete graph, Regular graph, Bipartite graph, Complete Bipartite graph with examples. **(07 Marks)**

c. Using the merge sort, sorts the list 6, 2, 7, 3, 4, 9, 5, 1, 8. **(07 Marks)**

OR

10 a. Define Isomorphic graphs with example and show that the following two graphs are isomorphic. **(10 Marks)**

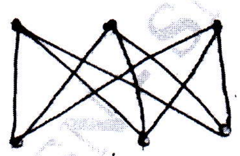

Fig. Q10 (a) (i)

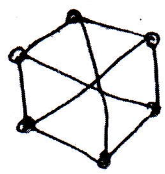

Fig. Q10 (a) (ii)

b. Obtain the Optimal prefix code for the message ROAD IS GOOD. Indicate the code. **(10 Marks)**

* * * * *